

Yo-Yo PHYSICS;
AN ENGINEER'S NOTEBOOK

THE SLEEPING Yo-YO

MONOGRAPH III
IN A SERIES

Don Watson

12/2000

Captain Yo

BERTRAND RUSSELL (1872-1970) FAMED MATHEMATICIAN AND PHILOSOPHER IS QUOTED:

"ONE OF THE SYMPTOMS OF AN APPROACHING NERVOUS BREAKDOWN IS THE BELIEF THAT ONE'S WORK IS TERRIBLY IMPORTANT."

CAPTAIN YO (1924-) SAYS:

"WRITING MONOGRAPHS HAS BEEN FUN. TIME NOW I GET BACK TO SERIOUS YO-YO PLAY..."

~~DU
12/100~~

HIGHLY RECOMMENDED:

THE BOOK OF YO, NEIL FESEN, INSIGHT PRESENTATIONS, P.O. BOX 540367, OMAHA NE, 68154. 1999

THE YONOMICON, MARK McBRIDE, INFINITE ILLUSIONS, TALLAHASSEE FL, 32303. 1998
Yo-Yo Times newsletter, P.O. Box 1519, HERNDON VA, 20172.

Yo-Yo PHYSICS:
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MONOGRAPH III
IN A SERIES

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INTRODUCTION

MATHEMATICAL MODELS BASED ON PHYSICAL ANALYSIS CAN BE FUN FOR THE CURIOUS STUDENT INVESTIGATOR - ESPECIALLY SO WHERE THE YO-YO IS THE SUBJECT. HERE, THE MODEL DEFINES THE SPIN DURATION AND SPIN VELOCITY (RPM) RELATIONSHIP FOR THE 'SLEEPING' YO-YO. IT PROVIDES AN EXCELLENT APPLIED SCIENCE EXERCISE, WELL-SUITED FOR HIGH SCHOOL AND UNIVERSITY LEVEL STUDIES - A VALUABLE TOOL FOR EDUCATORS AND STUDENTS OF THE PHYSICAL SCIENCES. THIS MONOGRAPH MAY ALSO BE USEFUL TO YO-YO DESIGNERS AND MANUFACTURERS, NOT TO EXCLUDE GROWING NUMBERS OF CREATIVE CRAFTERS, MODIFIERS, AND TINKERERS IN PRIVATE ACTIVITY.

"RADIUS OF GYRATION" AND "THE ACADEMIC YO-YO" - MONOGRAPHS I AND II IN THIS SERIES - DEVELOPED ANALYTIC AND EXPERIMENTAL METHODS TO PREDICT AND MEASURE THE MOMENT OF INERTIA ($I, \text{kg}\cdot\text{m}^2$) FOR ANY YO-YO. THOSE EFFORTS WERE PREREQUISITE TO THIS ONE; THE MOMENT OF INERTIA IS AN ESSENTIAL BEGINNING FACTOR IN THE SPIN DURATION AND INITIAL SPIN VELOCITY STUDY.

FRICITION AT THE YO-YO AXLE AND RESULTING RETARDING TORQUE CAN BE DIRECTLY QUANTIFIED IN FIXED AXLE DESIGNS, AND INDIRECTLY IN TRANSAXLE DESIGNS. IN FIXED AXLE CASES,

"THE CAPSTAN EQUATION" DIRECTLY APPLIES (PAGE 9). FOR THOSE INTERESTED AND WITH A KNOWLEDGE OF THE CALCULUS, THE EQUATION DERIVATION (ADAPTED FROM J. P. DEN HARTOG) BEGINS ON PAGE 31.

IN THE MODEL MATHEMATICS, UNITS OF MEASURE ARE SHOWN WHERE HELPFUL IN DEFINING GIVEN NUMBERS. LINE-OUT CANCELLATIONS OF UNITS ARE SHOWN WHERE APPROPRIATE FOR CLARITY AND ACCURACY IN PRESENTATION OF RESULTS. CORRECT MANIPULATION OF UNITS IS A MOST IMPORTANT TECHNIQUE IN MATHEMATICS - NOT TO BE SHORT-CUTTED. FOR THOSE UNFAMILIAR WITH THE NEWTON AND THE RADIAN, DETAILED DEFINITIONS BEGIN AT PAGE 36.

THERE IS MORE. SOME NINE PAGES OF TEXT AND SKETCHES ARE DEVOTED TO THE SUBJECT OF COMMON YO-YO STRING. THE INFORMATION GIVEN IS HARDLY COMPLETE, BUT IS IMPORTANT IN THE CONTEXT OF YO-YO SNAG-FREE SPIN DURATION, SPIN VELOCITY, AND RETURN RESPONSE.

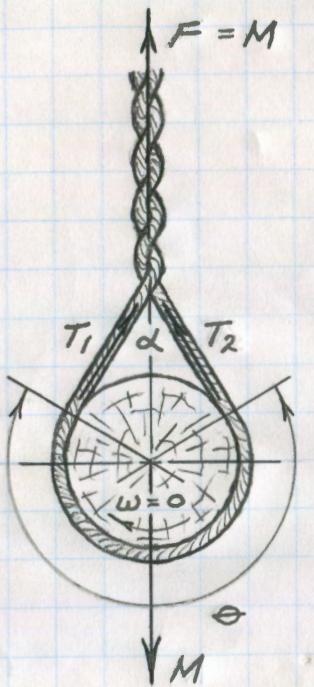
REMARKABLE ADVANCES IN YO-YO DESIGN AND PERFORMANCE HAVE EMERGED IN THE PAST DECADE, BUT PUBLISHED TECHNICAL LITERATURE REMAINS SCARCE TO NON-EXISTENT. THIS MONOGRAPH SERIES, "YO-YO PHYSICS", MAY BE A SMALL START IN FILLING THAT VOID; AND IT MAY OFFER A WHOLE NEW WAY TO HAVE FUN IN MATH AND PHYSICS - WITH THE YO-YO.

THE YO-YO AT REST

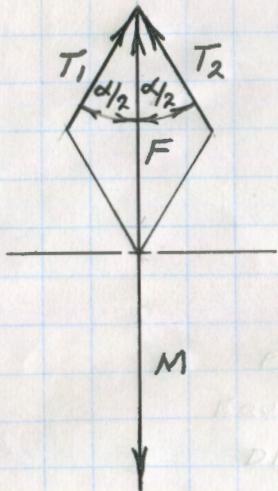
PLAYERS OF FIXED AXLE YO-YOS KNOW, OR SOON LEARN, THAT CONTROL OF THE YO-YO STRING IS THE KEY TO GOOD PERFORMANCE. SPECIFICALLY, THE TIGHTNESS OF THE LOOP STRAND ABOUT THE AXLE MUST BE NEITHER TOO TIGHT NOR TOO LOOSE. THE DEGREE OF TWIST IN THE DOUBLE STRAND YO-YO STRING BETWEEN THE HAND AND AXLE LOOP MUST NOT 'CHOKE' THE AXLE LOOP SO MUCH THAT THE YO-YO CANNOT SLEEP, BUT BE CHOKE'D AT LEAST ENOUGH THAT AXLE TO LOOP FRICTION WILL CAUSE THE YO-YO TO RETURN ON THE PLAYER'S DEMAND.

ALLOWING THE YO-YO TO HANG, NOT SPINNING, AT THE END OF THE VERTICAL STRING OFFERS AN OPPORTUNITY FOR EASY ANALYSIS REVEALING A REMARKABLE SENSITIVITY OF LOOP STRAND TENSIONS TO THE DEGREE OF CHOKE.

FIRST, THE STRAND TENSIONS OUTSIDE OF THE ARC OF CONTACT ARE EQUAL AND VARY DIRECTLY WITH THE YO-YO WEIGHT. SECOND, THESE SEGMENT TENSIONS BEAR A STRONG NON-LINEAR RELATIONSHIP TO THEIR INCLUDED ANGLE; THAT ANGLE BEING THE DIRECT MEASURE FOR THE DEGREE OF 'CHOKE' IN THE LOOP ABOUT THE AXLE.



THE
YO-YO
AT REST



FORCE
VECTOR
DIAGRAM

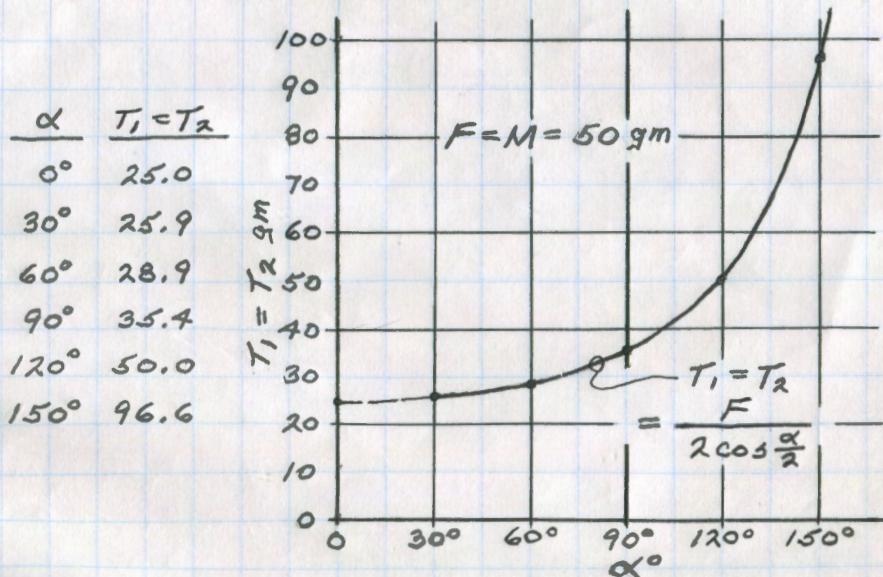
AXLE LOOP TIGHTNESS

THE YO-YO OF WEIGHT M gm SUSPENDED AT REST (SEE LEFT) EXERTS A TENSION F gm IN THE STRING. WITH THE YO-YO NOT ROTATING ($\omega = 0$), F HAS EQUAL COMPONENTS IN THE STRAND LOOPED AT THE AXLE ($T_1 = T_2$). INSPECTION OF THE VECTOR DIAGRAM GIVES:

$$F = T_1 \cos \frac{\alpha}{2} + T_2 \cos \frac{\alpha}{2}$$

$$T_1 = T_2 = \frac{F}{2 \cos \frac{\alpha}{2}}$$

THIS RELATIONSHIP, FOR A TYPICAL YO-YO WEIGHT $M = 50$ gm, DEVELOPS A USEFUL TABLE AND GRAPH:



WHERE α EXCEEDS 90° , AXLE LOOP TENSIONS T_1 AND T_2 ARE EXTREMELY SENSITIVE TO CHANGES IN α . IN FIXED AXLE YO-YOS WITH α GREATER THAN 90° , LOSS OF 'SLEEP' TIME IS SIGNIFICANT, WHILE COMPLETE FAILURE TO 'SLEEP' IS MOST LIKELY. WHERE α IS LESS THAN 45° , FIXED AXLE YO-YOS MAY 'SLEEP' LONG AND RELIABLY, BUT FAIL RETRIEVAL ATTEMPTS WITH AGGRAVATING CONSISTENCY.

EACH OVERHAND THROW TO A 'SLEEPER' RESULTS IN A HALF-TWIST OF THE STRING WHEN THE PALM IS TURNED DOWN TO RETRIEVE THE YO-YO. FOR RIGHT-HANDERS EACH THROW INCREMENTALLY TIGHTENS THE AXLE LOOP AND INCREASES THE ANGLE α . FOR LEFT-HANDERS THE LOOP IS LOOSENED AND α IS DECREASED. THESE EFFECTS MUST BE CORRECTED PERIODICALLY BY THE PLAYER TO MAINTAIN THE AXLE LOOP TIGHTNESS IN A RANGE OF RELIABLE YO-YO PLAY.

THE YO-YO AT REST SKETCH SHOWS THAT THE YO-YO SPUN COUNTER-CLOCKWISE (VIEWED FROM ABOVE) LOOSENS THE STRING TWIST AND AXLE LOOP; i.e. "LEFTY-YOOSY!" THE YO-YO SPUN CLOCKWISE TIGHTENS BOTH; i.e. "RIGHTY-TIGHTY!"

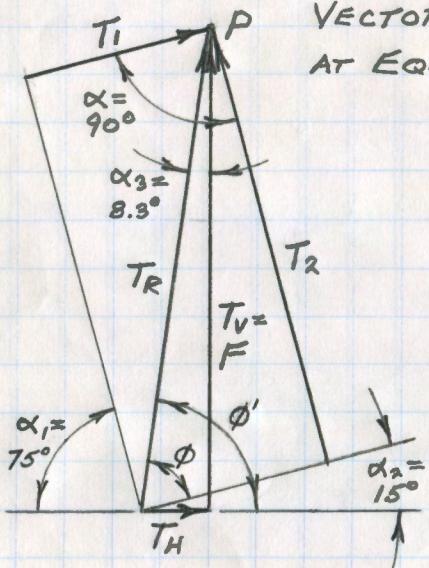
NOVICES BEWARE. FIXED AXLE YO-YO PLAY REQUIRES THAT STRING TWIST AND AXLE LOOP TIGHTNESS BE CORRECTED OFTEN.

THE SLEEPING YO-YO

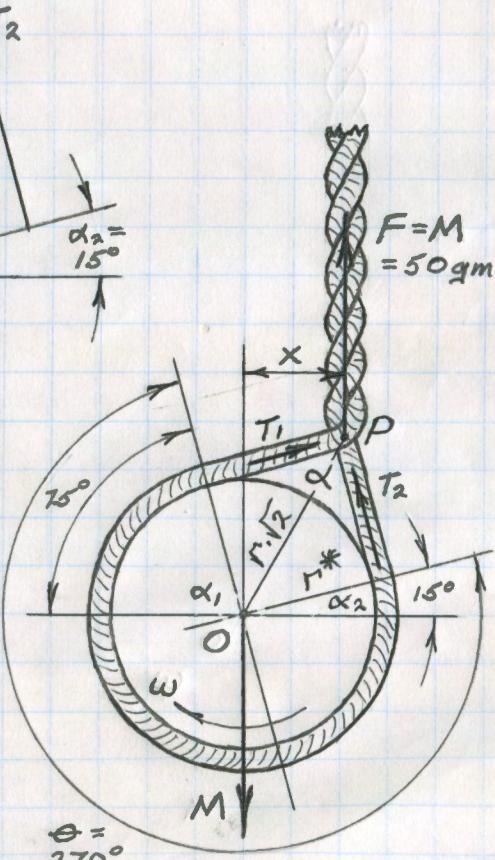
ANY SLEEPING FIXED AXLE YO-YO PRODUCES 'CAPSTAN' FRICTION AT THE AXLE AND STRING LOOP ARC OF CONTACT. SIGHTING VERTICALLY DOWN THE STRING WILL SHOW THAT CLOCKWISE SPIN DEFLECTS THE STRING LINE TO THE RIGHT OF THE AXLE CENTER; COUNTER-CLOCKWISE SPIN DEFLECTS THE STRING LINE TO THE LEFT.

THE CAPSTAN EQUATION (SEE CONTENTS) OFFERS INSIGHT INTO THE PHYSICAL MECHANICS OF THE SLEEPING YO-YO AT EQUILIBRIUM. THE AXLE ANGULAR VELOCITY IN THE STRING LOOP IS NOT A FACTOR SO LONG AS IT IS NOT ZERO.

THE FOLLOWING PAGE DISPLAYS A TYPICAL RELATIONSHIP FOR A SLEEPING YO-YO WITH ITS SPINNING BIRCH WOOD AXLE IN THE COTTON STRING LOOP (LOWER RIGHT). ALSO SHOWN IS A COMPANION DIAGRAM OF VECTOR FORCES FOR THE YO-YO SPINNING IN EQUILIBRIUM CONDITION. EQUILIBRIUM REQUIRES THAT OPPOSING FORCES ON THE YO-YO BE BALANCED; THAT THE YO-YO, THOUGH SPINNING, BE FIXED IN POSITION. AT ANY INSTANT, OPPOSING TORQUES MUST ALSO BE BALANCED. AT THE EQUILIBRIUM STATE, THE STATIC FORCES ON THE HANGING YO-YO AND THE CAPSTAN FRICTION INDUCED TORQUES CAN BE RESOLVED IN A REALISTIC PHYSICAL ANALYSIS.



VECTOR FORCES AT EQUILIBRIUM



* IN FIXED AXLE

YO-YOS, $m \approx 0.125 \text{ in}$

THE SLEEPING YO-YO

IN THE STRAND OF STRING LOOPED ABOUT THE SPINNING AXLE (SEE LEFT), THE CAPSTAN EQUATION APPLIES:

$$\frac{T_2}{T_1} = e^{u\theta} \text{ OR } T_2 = T_1 e^{u\theta}$$

WHERE $e = 2.7183$ (THE BASE OF NATURAL LOGARITHMS) AND $u = 0.23^*$ (HERE, THE COEFFICIENT OF FRICTION FOR BIRCH TO COTTON). FOR AN ASSUMED ANGLE $\alpha = 90^\circ$ BETWEEN THE FREE STRANDS, $\theta = 4.71$ RADIANS (AT 2π RADIANS PER 360°). ASSUME ALSO THAT $\alpha_1 = 75^\circ$ AND $\alpha_2 = 15^\circ$. INSPECTION OF THE PICTORIAL YIELDS:

$$T_1 \cos \alpha_1 + T_2 \cos \alpha_2 = F$$

$$T_1 \cos \alpha_1 + T_1 e^{u\theta} \cos \alpha_2 = F$$

$$T_1 = \frac{F}{\cos \alpha_1 + e^{u\theta} \cos \alpha_2} = \frac{50}{0.259 + (2.324 \cdot 0.966)}$$

$$\underline{T_1 = 20.0 \text{ gm}}; \quad T_2 = 20.0 \cdot 2.324$$

$$\underline{T_2 = 46.5 \text{ gm}}$$

* APPROXIMATE MEASURE BY TWO METHODS;
(INCLINE AND HORIZONTAL) ACROSS THE WOOD GRAIN USING A SLAB OF BIRCH VENEER, LEVEL, CALIPER, AND SUITABLE SPRING SCALES.

FULL DEFINITION OF THE VECTOR FORCES AT EQUILIBRIUM DIAGRAM CAN BE GAINED WITH T_R , THE VECTOR RESULTANT OF KNOWN VECTOR TENSIONS T_1 AND T_2 . FROM T_R , ITS VERTICAL AND HORIZONTAL COMPONENTS $T_V (=F)$ AND T_H , ϕ (ANGLE OF T_R WITH T_1), ϕ' AND α_3 (ANGLES OF T_R WITH THE HORIZONTAL AND VERTICAL) ARE READILY DETERMINED:

$$T_R = \sqrt{T_1^2 + T_2^2} = (20.0^2 + 46.5^2)^{\frac{1}{2}} = 2562^{\frac{1}{2}}$$

$$\underline{T_R = 50.6 \text{ gm}}$$

$$\phi = \cos^{-1} \frac{20.0}{50.6} = \cos^{-1} 0.395$$

$$\underline{\phi = 66.7^\circ}$$

$$\phi' = \phi + \alpha_2 = 66.7 + 15.0$$

$$\underline{\phi' = 81.7^\circ} ; \underline{\alpha_3 = 8.3^\circ}$$

$$T_V = T_R \sin 81.7^\circ = 50.6 \cdot 0.990$$

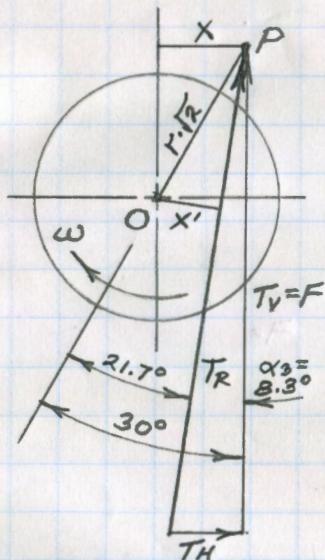
$$\underline{T_V = 50.0 \text{ gm} (=F = M)}$$

$$T_H = T_R \cos 81.7^\circ = 50.6 \cdot 0.147$$

$$\underline{T_H = 7.3 \text{ gm}}$$

CLOCKWISE DRIVING TORQUE T_c ABOUT THE AXLE CENTER (O) IS EXERTED AT THE AXLE TO STRING ARC OF CONTACT BY THE YO-YO SPINNING MOMENTUM. EQUILIBRIUM OCCURS AS THE VERTICAL STRING LINE MOVES RIGHT (X) AND THE YO-YO CLIMBS THE TIGHT SIDE OF THE STRING LOOP.

T_R ACTS UPWARD THROUGH POINT P AT AN ANGLE α_3 AND RIGHT OF THE AXLE CENTER (O) A



NORMAL DISTANCE x' , EXERTING A COUNTER-CLOCKWISE TORQUE T_{cc} ABOUT THE AXLE CENTER. THIS DIAGRAM IS TAKEN FROM THE EARLIER VECTOR FORCES AT EQUILIBRIUM DIAGRAM. HERE:

$$\begin{aligned} x' &= r \cdot \sqrt{2} \cdot \sin 21.7^\circ \\ &= 0.125 \cdot 1.414 \cdot 0.370 \\ x' &= 0.065 \text{ in} \end{aligned}$$

COUNTER-CLOCKWISE TORQUE IS:

$$\begin{aligned} T_{cc} &= T_R \cdot x' = 50.6 \cdot 0.065 \\ T_{cc} &= 3.3 \text{ gm-in} \end{aligned}$$

CAPSTAN CLOCKWISE (NET) TORQUE IS:

$$\begin{aligned} T_c &= (T_2 - T_1) \cdot r = (46.5 - 20.0) \cdot 0.125 \\ T_c &= 3.3 \text{ gm-in} \end{aligned}$$

CAPSTAN FRICTION BRINGS THE SLEEPING YO-YO TO A POSITION OF EQUILIBRIUM, ELEVATED SLIGHTLY FROM THE YO-YO AT REST POSITION, AND WITH THE SUSPENDING STRING OFFSET IN THE DIRECTION OF SPIN. THE EQUILIBRIUM POSITION IS MAINTAINED SO LONG AS STRING TENSION F AND YO-YO WEIGHT M REMAIN EQUAL, AND SO LONG AS THE CAPSTAN FRICTION AND STRING TENSION INDUCED TORQUES T_C AND T_{CC} REMAIN EQUAL.

AS THE STORED ENERGY OF THE SPINNING YO-YO IS LOST TO WORK DONE IN FRICTION AT THE AXLE TO STRING ARC OF CONTACT, THE ANGULAR VELOCITY ω DECAYS. IN THAT PERIOD OF DECAY, THE EQUILIBRIUM POSITION REMAINS FIXED; IN A BRIEF INTERVAL AS THE ANGULAR VELOCITY APPROACHES AND REACHES ZERO, THE YO-YO FALLS TO THE YO-YO AT REST POSITION.

THE ANALYSIS PRESENTED HERE IS BY NO MEANS PRECISE IN EITHER ASSUMPTIONS OR CALCULATED RESULTS. NEVERTHELESS, IT DOES OFFER A REASONABLY ACCURATE (AND, IT IS HOPE, INTERESTING) EVALUATION OF QUANTITATIVE PHYSICS FOR THE SLEEPING YO-YO IN A SPECIFIC "SNAPSHOT" STATE OF EQUILIBRIUM - POISED AND READY TO RETURN TO THE PLAYER'S HAND WITH A BRIEF VERTICAL TUG ON THE STRING.

BEYOND THE BRIEF TUG-INDUCED INCREASE IN STRING TENSION, UPWARD YO-YO MOMENTUM INDUCES SLACK IN THE STRING. THE AXLE SURFACE AND SPINNING SIDES CAN ENGAGE THE SLACKENED STRING, WRAPPING IT AT THE AXLE, THEN WEDGING AND WINDING IT IN THE STRING GAP.

IN PRACTICE, SLACK IN THE STRING IS VERY OFTEN INDUCED WITHOUT TUGGING. GIVEN A SLEEPING YO-YO HANGING AT THE END OF THE STRING, INITIATE A RETURN BY:

1. GIVING THE YO-YO HAND A SHARP DOWNWARD SLAP WITH THE FREE HAND; HERE, THE VERTICAL INERTIA OF THE YO-YO ALLOWS THE DOWNWARD YO-YO HAND MOTION TO SLACKEN THE STRING. (TRICK NAME, 'BURP THE BABY')

2. SWINGING THE YO-YO FORWARD, BRINGING THE STRING LINE ABOVE THE HORIZONTAL; HERE LOSS OF SWING-INDUCED CENTRIFUGAL FORCE SLACKENS THE STRING WITH SOME HELP FROM GRAVITY. (TRICK TERMINATION FOR 'BREAKAWAY' AND 'AROUND THE WORLD')

SENSITIVITY OF THE SLEEPING FIXED AXLE YO-YO TO THE SLACKENED STRING PRESENTS PLAYERS WITH TECHNIQUES TO MASTER:

- THROWING A 'SLEEPER' REQUIRES A FINESSE IN ARM, HAND, AND FINGER ACTION TO AVOID BOUNCING THE YO-YO OFF THE END OF THE STRING; BOUNCE-INDUCED SLACK WILL PROBABLY RETURN THE YO-YO. A HARDER

THROW AMPLIFIES THE BOUNCE, MAKING FAILURE TO SLEEP EVEN MORE LIKELY.

- IN SLEEPING TRICKS, TENSION MUST BE MAINTAINED IN THE STRING LEAD TO THE SPINNING AXLE; ABRUPT TENSION CHANGES MUST BE AVOIDED.

A LESSER KNOWN PROBLEM RELATED TO STRING TENSION IN SLEEPING TRICKS OCCURS WHERE PLAY IS ATTEMPTED IN A SIDE BREEZE. THE BREEZE CAN BLOW A NEARLY SLACK STRING TO ONE SIDE - GENTLY INDUCING AND MAINTAINING ENOUGH STRING TENSION TO PREVENT RETURN OF THE YO-YO. IN CONTESTS AND IN DEMONSTRATIONS, KNOWLEDGEABLE PLAYERS PERFORM WITH BREEZE AT THE BACK.

WHAT HAS BEEN OFFERED HERE IS A BIT OF QUALITATIVE APPLIED PHYSICS - A BRIEF TWO-PAGE RESPITE FROM THE PRECEDING QUANTITATIVE ANALYSIS WITH NUMBERS AND VECTORS.

YO-YO PLAYERS HAVE LITTLE NEED FOR THE QUANTITATIVE OR THE QUALITATIVE STUDIES. THEIR APPLICATION OF BODY-HAND-EYE COORDINATION, SPLIT-SECOND REACTION, IMAGINATION, CREATIVITY, AND (ESPECIALLY) INTUITION, WITH MUCH PRACTICE AND DEDICATION, LEADS THEM TO DAZZLING DISPLAYS OF 'APPLIED PHYSICS' IN THE TRUEST SENSE OF THE PHRASE. HERE LIES ONE OF THE MANY BEAUTIES OF THE YO-YO MYSTIQUE. THEY 'JUST DO IT!'.

SPIN VELOCITY

IN MONOGRAPH I - RADIUS OF GYRATION, THE MOMENT OF INERTIA I FOR A POPULAR 'CONTEMPORARY YO-YO' WAS EVALUATED.

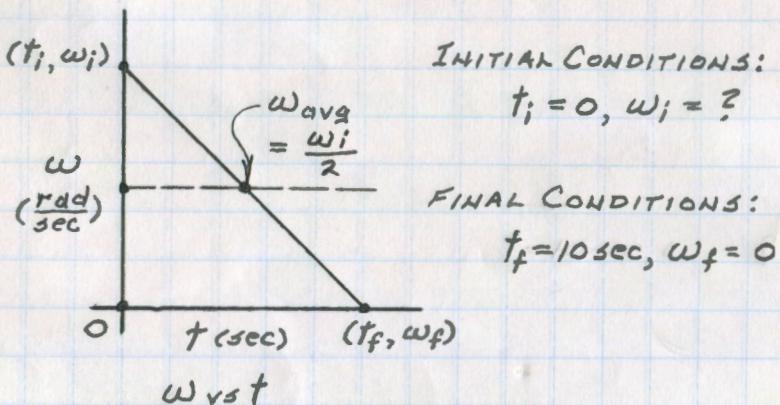
IN THE SLEEPING YO-YO ANALYSIS (AT EQUILIBRIUM) THE RETARDING TORQUE T_{cc} FOR THE SAME YO-YO WAS DETERMINED; IT WAS ALSO STATED THAT THE ANGULAR VELOCITY ω ($\neq 0$) IS NOT A FACTOR IN THE CAPSTAN FRICTION DEVELOPED AT THE FIXED AXLE. IT FOLLOWS THAT RETARDING TORQUE IS CONSTANT AND THE ANGULAR VELOCITY DECAYS IN LINEAR RELATIONSHIP WITH THE ELAPSED TIME t ; THAT IS TO SAY THAT THE NEGATIVE ANGULAR ACCELERATION α IS A CONSTANT.

THE PHYSICS REFERENCES LEAD TO:

$$\tau = J\alpha \text{ OR } \alpha = \frac{\tau}{J} \frac{\text{rad}}{\text{sec}^2}$$

THIS RELATIONSHIP REQUIRES THAT T_{cc} BE THE PRODUCT OF FORCE EXPRESSED IN NEWTONS (1 NEWTON = $\frac{\text{kg} \cdot \text{m}}{\text{sec}^2} = 0.225 \text{ lb.}$) AND THE RADIUS r (METERS) AT WHICH THAT FORCE ACTS. WITH J EXPRESSED IN $\text{kg} \cdot \text{m}^2$, α IS CALCULATED IN RADIANS/SEC 2 . ONCE α IS KNOWN, THE INITIAL ANGULAR VELOCITY ω_i IS ONLY A BRIEF STEP AWAY - GIVEN THE LINEAR RELATIONSHIP OF $\omega_i = \omega_0 + \alpha t$. A MATHEMATICAL MODEL USING

THESE CONDITIONS IS OFFERED HERE FOR A MODERATE 10 SEC DURATION 'SLEEPER' WITH THE 'CONTEMPORARY YO-YO' OF MONOGRAPH I. A GENERALIZED PLOT OF ω vs t PICTURES THE LINEAR RELATIONSHIP AND DEFINES THE INITIAL AND FINAL 'SLEEP' VALUES.



FOR THE 'CONTEMPORARY YO-YO':

$$I = 21600 \times 10^{-9} \text{ kg-m}^2$$

$$\text{AXLE RADIUS, } r = 0.130 \text{ in} \cdot 25.4 \times 10^{-3} \text{ m} = \underline{\underline{3.3 \times 10^{-3} \text{ m}}}$$

RETARDING FORCE AT RADIUS r , $F \approx \underline{\underline{(46.5 - 20.0) \text{ gm}}}$

$$F(\text{NEWTONS}) = 26.5 \text{ gm} \cdot \frac{1 \text{ N}}{453.6 \text{ gm}} \cdot \frac{1 \text{ N}}{0.225 \text{ m}} = \underline{\underline{0.26 \text{ N}}}$$

RETARDING TORQUE AT RADIUS r , $T_{cc} = \underline{\underline{F \cdot r}} \text{ N-m}$

$$T_{cc} \approx 0.26 \frac{\text{kg-m}}{\text{sec}^2} \cdot 3.3 \times 10^{-3} \text{ m} \approx \underline{\underline{858000 \times 10^{-9} \frac{\text{kg-m}^2}{\text{sec}^2}}}$$

NEGATIVE ACCELERATION, $\alpha = \frac{\frac{\text{rad}}{\text{sec}}}{\text{sec}} \frac{\text{rad}}{\text{sec}^2}$

$$\alpha \approx \frac{858000 \times 10^{-9}}{21600 \times 10^{-9}} \frac{\frac{\text{kg} \cdot \text{m}^2}{\text{sec}^2}}{\text{kg} \cdot \text{m}^2} \approx \frac{40}{\text{sec}^2} \frac{\text{rad}}{\text{sec}^2}$$

NOTE THAT WHERE THE UNITS $\text{kg} \cdot \text{m}^2$ CANCEL,
 α IS LEFT WITH (DIMENSIONLESS) RADIANS AS
 THE NUMERATOR AND sec^2 AS THE DENOMINATOR.

FROM THE LINEAR W VS t :

$$w_f = w_i - (\alpha t_f) = 0$$

GIVEN $t_f = 10 \text{ sec.}^*$, THE INITIAL ANGULAR
 VELOCITY w_i BECOMES:

$$w_i = \alpha t_f \approx 40 \frac{\text{rad}}{\text{sec}^2} \cdot 10 \text{ sec}$$

$$w_i = 400 \frac{\text{rad}}{\text{sec}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \cdot \frac{60 \text{ sec}}{\text{min}} \approx \underline{\underline{3820 \text{ RPM}}}$$

'SLEEP' REVOLUTIONS, $\theta = w_{avg} \cdot t = \frac{w_i + w_f}{2} \cdot t \text{ rev}$

$$\theta \approx \frac{3820 + 0}{2} \frac{\text{rev}}{\text{min}} \cdot 10 \text{ sec} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$$

$$\theta \approx \underline{\underline{318 \text{ rev}}}$$

* ACCURACY IN SLEEP DURATION TO $w_f = 0 \text{ RPM}$
 REQUIRES CONTROL OF YO-YO PRECESSION TO
 AVOID CONTACT BETWEEN THE STRING AND THE
 SPINNING SIDES.

MONOGRAPH I FOUND THE 'CONTEMPORARY YO-YO' AND THE 'MODERN TRANSAXLE YO-YO' TO BE OF SIMILAR WEIGHT AND NEARLY EQUAL MOMENT OF INERTIA. REASONABLE INFERENCE MIGHT ALLOW THAT, FOR THROWS OF 'EQUAL VIGOR', THE YO-YOS REACH THE SAME INITIAL 'SLEEP' ANGULAR VELOCITY ω_i ; FOR A SECOND MATHEMATICAL MODEL, NOMINALLY 3820 RPM.

THE TRANSAXLE BALL BEARING, PROVIDING IT IS FREE OF SIGNIFICANTLY VISCOUS LUBRICANT, EXHIBITS VERY LOW ROLLING FRICTION. HERE, AS IN THE FIXED AXLE CASE, THE ANGULAR VELOCITY ω ($\neq 0$) DECAYS IN LINEAR FASHION IN THE SLEEP DURATION t , AND THE NEGATIVE ANGULAR ACCELERATION α CAN BE ASSUMED CONSTANT.

SUBJECTIVELY 'EQUAL VIGOR' THROWS OF THE TRANSAXLE YO-YO YIELD SPIN DURATIONS $t \approx 60$ SEC. FOR THE BALL BEARING, AN EFFECTIVE COEFFICIENT OF FRICTION μ_e IS ESTIMATED IN THIS SECOND MATHEMATICAL MODEL, REVERSING THE SEQUENCE OF CALCULATION WHERE IN ORDER OF USE:

- $\omega_i \approx 3820$ RPM, $\omega_f = 0$
- $t \approx 60$ SEC
- $J \approx 22100 \times 10^{-9}$ $\text{kg}\cdot\text{m}^2$
- $r = 0.110$ in (BALL \varnothing)
- $M = 47.3$ gm

FOR THE 'MODERN TRANSAXLE YO-YO':

$$'SLEEP' REVOLUTIONS \theta = \frac{\omega_i + \omega_f}{2} \cdot t$$

$$\theta \approx \frac{3820+0}{2} \frac{\text{rev}}{\text{min}} \cdot 60 \text{ sec} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$$

$$\theta \approx \underline{\underline{1910 \text{ rev}}}$$

$$\alpha = \frac{\omega_i}{t} \approx 3820 \frac{\text{rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot 2\pi \frac{\text{rad}}{\text{rev}} \div 60 \text{ sec}$$

$$\alpha \approx \underline{\underline{6.67 \frac{\text{rad}}{\text{sec}^2}}}$$

$$J_{cc} = I \cdot \alpha \approx 22100 \times 10^{-9} \text{ kg-m}^2 \cdot 6.67 \frac{\text{rad}}{\text{sec}^2}$$

$$J_{cc} \approx \underline{\underline{147400 \times 10^{-9} \frac{\text{kg-m}^2}{\text{sec}^2}}} \text{ OR N-m}$$

$$r = 0.110 \text{ in} \cdot 25.4 \times 10^{-3} \frac{\text{m}}{\text{in}}$$

$$r = \underline{\underline{2.8 \times 10^{-3} \text{ m}}}$$

$$F(\text{NEWTONS}) = \frac{J_{cc}}{r} \frac{\frac{\text{kg-m}^2}{\text{sec}^2}}{\text{m}} \text{ OR N}$$

$$F \approx \frac{147400 \times 10^{-9}}{2.8 \times 10^{-3}} \frac{\text{N-m}}{\text{m}} \cdot 0.225 \frac{\text{lb}}{\text{in}} \cdot 453.6 \frac{\text{gm}}{\text{lb}}$$

$$F \approx \underline{\underline{5.37 \text{ gm}}}$$

$$u_e \approx \frac{F}{M} \approx \frac{5.37}{47.3} \frac{\text{gm}}{\text{gm}}$$

$$u_e \approx \underline{\underline{0.114}}, \text{ FOR 'ROLLING' FRICTION}$$

MODEL SUMMARY

<u>FACTOR, UNITS</u>	<u>CONTEMPORARY TRANSAXLE</u>		<u>MODERN</u>
	<u>Yo-Yo</u>	<u>Yo-Yo</u>	
MOMENT OF INERTIA, $\sim \text{kg-m}^2$	$^3 21600 \times 10^{-9}$	$^3 22100 \times 10^{-9}$	
WEIGHT, M gm	$^2 49.4$	$^2 47.3$	
AXLE RADIUS, R in	$^2 0.130$	$^2 0.110$	
COEFFICIENT OF FRICTION, ATM, u	$^2 0.230$	$^3 0.114$	
FORCE AT R, F gm	26.5	5.37	
RETARDING TORQUE, $T_{CC} \frac{\text{kg-m}^2}{\text{sec}^2} \text{OKN-m}$	$^3 858000 \times 10^{-9}$	$^3 147400 \times 10^{-9}$	
NEGATIVE ACCELERATION, $\propto \frac{\text{rad}}{\text{sec}^2}$	$^3 40$	$^3 6.67$	
INIT. ANGULAR VELOCITY, W; RPM	$^3 3820$	$^1 3820$	
'SLEEP' REVOLUTIONS, Θ rev	$^3 318$	$^3 1910$	
SPIN DURATION, t sec	$^2 10$	$^2 60$	

SOURCES: ¹ ASSUMED ² MEASURED ³ CALCULATED

CAUTION REASONABLE ANALYTIC METHODS
LEAD TO ONLY REASONABLE MODEL DATA. MORE
ACCURATE DATA REQUIRES BETTER MEASURE-
MENTS AND FURTHER EXPERIMENT - BEYOND THIS
AUTHOR'S RESOURCES, AND BEYOND THE SCOPE
OF THESE INITIAL INVESTIGATIONS.

SPIN ENERGY

INITIAL 'SLEEP' KINETIC ENERGY AND ITS DISSIPATION CAN BE ASSESSED FROM THE MODEL SUMMARY DATA. FOR THE CONTEMPORARY YO-YO:

$$\omega_i = 3820 \frac{\text{rev}}{\text{min}} \cdot \frac{1 \text{min}}{60 \text{sec}} \cdot \frac{2\pi \text{rad}}{\text{rev}} = \underline{\underline{400}} \frac{\text{rad}}{\text{sec}}$$

$$\begin{aligned} \text{K.E.} &= \frac{1}{2} I \omega_i^2 = \frac{1}{2} \cdot 21600 \times 10^9 \frac{\text{kg} \cdot \text{m}^2}{\text{sec}^2} \cdot (4 \times 10^2)^2 \frac{\text{rad}^2}{\text{sec}^2} \\ &= 172800 \times 10^{-5} \frac{\text{kg} \cdot \text{m}^2}{\text{sec}^2} = 1.728 \frac{\text{kg} \cdot \text{m}^2}{\text{sec}^2} \text{ OR N-m} \\ &= 1.728 \text{ N-m} \cdot 0.225 \frac{\text{J}}{\text{N}} \cdot 453.6 \times 10^{-3} \frac{\text{kg}}{\text{J}} \end{aligned}$$

$$\text{K.E.} = \underline{\underline{176.4 \times 10^{-3}}} \text{ kg-m}$$

SPIN ENERGY IS LOST IN WORK DONE:

RETARDING FORCE AT RADIUS r , $F \approx 26.5 \text{ gm}$

$$F = 26.5 \text{ gm} \times 10^{-3} \frac{\text{kg}}{\text{gm}} = \underline{\underline{26.5 \times 10^{-3}}} \text{ kg}$$

AXLE CIRCUMFERENCE AT r , $C = 2\pi r$

$$C = 2\pi \cdot 0.130 \frac{\text{m}}{\text{in}} \cdot 25.4 \times 10^{-3} \frac{\text{m}}{\text{in}} = \underline{\underline{20.75 \times 10^{-3}}} \frac{\text{m}}{\text{rev}}$$

SPIN DURATION WORK DONE, $W = F \cdot C$

$$W = 26.5 \times 10^{-3} \text{ kg} \cdot 20.75 \times 10^{-3} \frac{\text{m}}{\text{rev}} \approx \underline{\underline{550 \times 10^{-6}}} \frac{\text{kg-m}}{\text{rev}}$$

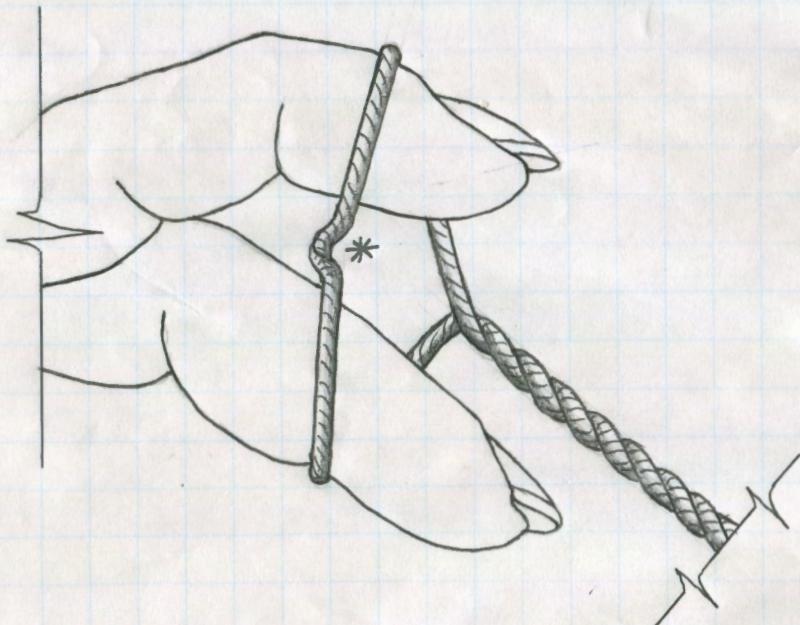
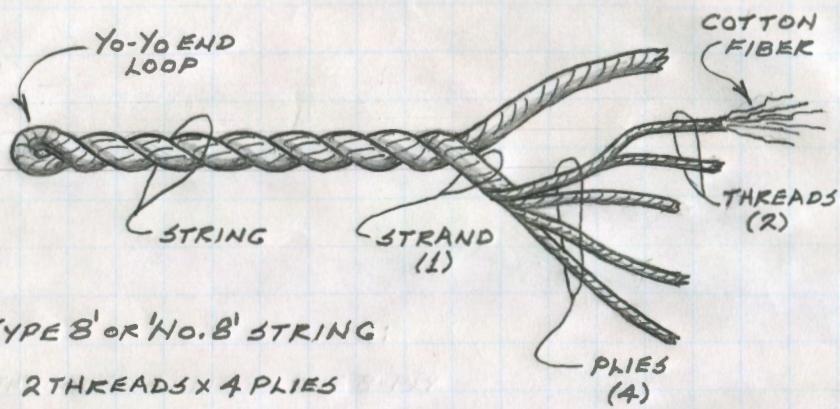
SPIN REVOLUTIONS θ , AND SPIN DURATION t :

$$\theta = \frac{\text{K.E.}}{W} = \frac{176.4 \times 10^{-3}}{550 \times 10^{-6}} \frac{\text{kg-m}}{\text{kg-m}} \approx \underline{\underline{320}} \text{ rev}$$

$$t = \frac{\theta}{\omega_{\text{avg}}} = \theta \text{ rev} / \frac{(w_i + w_0)}{2} \frac{\text{rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$$

$$= 320 \text{ rev} / \frac{(3820 + 0)}{120} \frac{\text{rev}}{\text{sec}}$$

$$t \approx \underline{\underline{10}} \text{ sec}$$



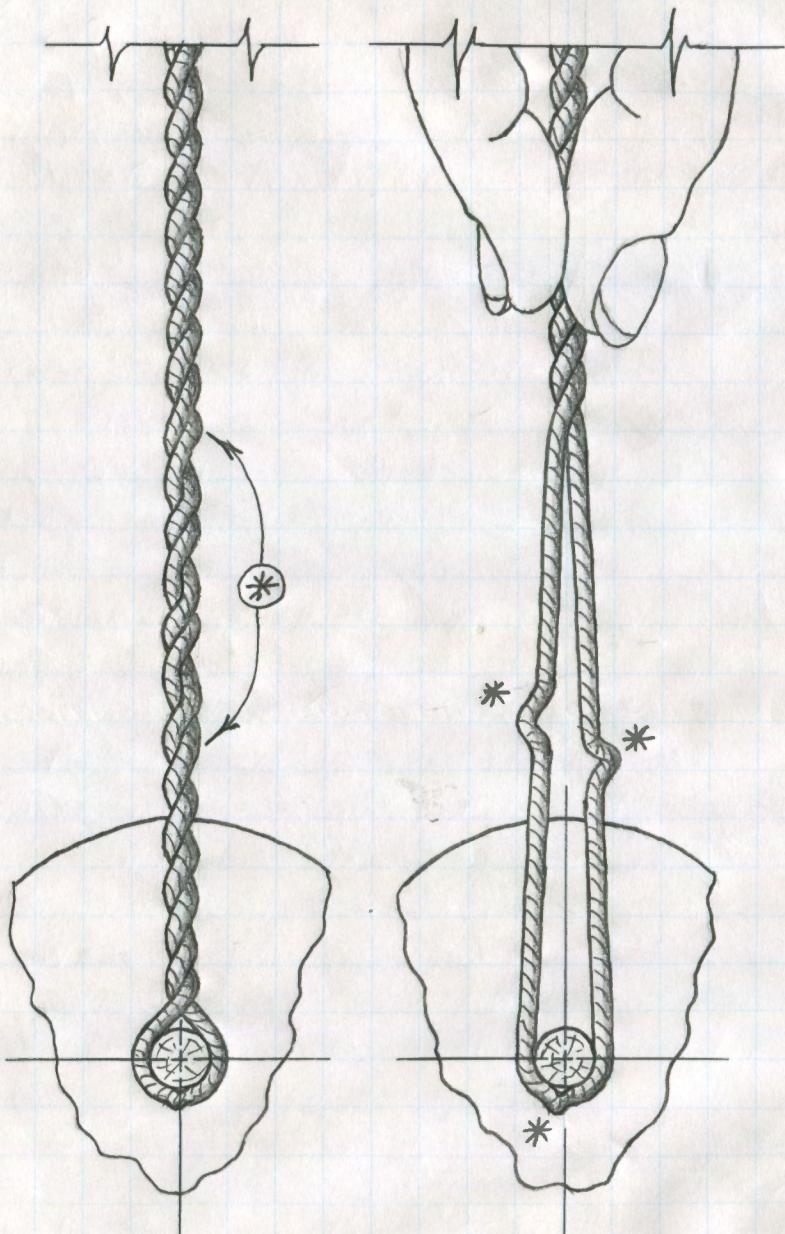
* AXLE LOOP KINK

THE STRING'S THE THING

THE STRING IS THE Yo-YO PLAYER'S INDISPENSIBLE TOOL. EARLIER, (SEE AXLE LOOP TIGHTNESS) THE NEED TO ADJUST STRING TWIST AND LOOP TIGHTNESS WAS EMPHASIZED. WHEN THE STRING IS ATTACHED TO THE Yo-YO, KINKS COMMONLY FOUND IN THE SINGLE STRAND MUST BE ELIMINATED OR MINIMIZED IF CONSISTENTLY GOOD PERFORMANCE IS TO BE ACHIEVED.

AT THE LEFT, A SHORT SECTION OF 'TYPE 8' OR 'No. 8' STRING IS PICTURED SHOWING COTTON FIBER SPUN INTO A THREAD. TWO SUCH THREADS ARE TWISTED TO FORM A PLY, AND FOUR SUCH PLIES ARE TWISTED TO FORM A SINGLE STRAND. THE 8-THREAD STRAND IS FOLDED BACK ON ITSELF (FORMING THE END LOOP) WITH THE DOUBLED STRAND THEN TWISTED AND TIED WITH A FINGER LOOP TO COMPLETE THE STRING, TYPICALLY AT A 40-INCH LENGTH.

THE OPENED END LOOP ON THE FINGERS IN THE LOWER SKETCH REVEALS AN AXLE LOOP KINK*, ESPECIALLY PROMINENT WHEN STRAND-PLIES HAVE BEEN TWISTED TOO TIGHTLY IN PRODUCTION. SUCH KINKS AT THE AXLE LOOP INDUCE EXCESSIVE FRICTION WITH FIXED AXLES AND SPINNING SIDES. IN 'STRING' TRICKS, WITH THE DOUBLED STRAND STRING AND SINGLE STRAND AXLE LOOP SHARING THE STRING GAP, PERFORMANCE FAILURE



Ⓐ SLOPPY
TWIST

* STRAND
KINKS

IS THE LIKELY RESULT. AT BEST, SIGNIFICANT LOSS OF 'SLEEP' SPIN DURATION OCCURS.

'SLOPPY TWIST' (SEE LEFT ) MAY OCCUR ANYWHERE IN THE DOUBLED STRAND LENGTH; IT USUALLY OCCURS NEAR THE RIM OF THE YO-YO WHEN THE STRING IS ATTACHED. OPENING THE AXLE LOOP TO A POINT ABOVE THE 'SLOPPY TWIST' WILL, WITH A LITTLE SLACK, REVEAL TWO STRAND KINKS*.

THE FOLLOWING TECHNIQUE ELIMINATES THE SLOPPY TWIST AND THE TWO UPPER KINKS; IT ALSO MINIMIZES THE MORE CRITICAL AXLE LOOP KINK:

1. START WITH THE AXLE LOOP OPEN TO THE FINGERS AT LEAST TWO INCHES BEYOND THE YO-YO RIM; FARTHER WHEN NECESSARY.

2. INDUCE TENSION IN THE STRING WHILE MOVING THE FINGERS ALONG THE STRING - AWAY FROM THE YO-YO A FOOT OR TWO.

IN STEP 2, THE INDUCED TENSION FLATTENS ALL THREE STRAND KINKS. MOVING THE FINGERS AWAY FROM THE YO-YO ALLOWS THE DOUBLED STRAND TO REDISTRIBUTE THE TWIST EVENLY TOWARD THE AXLE, ELIMINATING THE UPPER KINKS. A FINAL FLIP OF THE YO-YO TO FURTHER TIGHTEN THE AXLE LOOP MINIMIZES THE AXLE LOOP KINK. BETTER YO-YO PERFORMANCE SHOULD BE THE RESULT.

STRING CHARACTERISTICS

AMONG THE MANY STRING VARIATIONS AVAILABLE ARE A 'TYPE 6' COTTON BLEND AND A 'TYPE 8' COTTON. SOME CHARACTERISTICS OF CONSTRUCTION AND USE ARE:

<u>CHARACTERISTIC</u>	Yo-Yo STRING	
	<u>TYPE 6</u>	<u>TYPE 8</u>
THREADS/PLY	2	2
PLIES/STRAND	3	4
THREADS/STRAND	6	8
STRAND DIAMETER ¹	$\approx 0.030"$	$\approx 0.035"$
STRING DIAMETER ²	$\approx 0.050"$	$\approx 0.060"$
WOUND RADIUS ² , r_3	$1\frac{1}{16}D \approx 0.53"$	$1\frac{3}{16}D \approx 0.59"$

- 1 APPROXIMATE MEASURE FOR NEW STRING UNDER MODERATE TENSION.
- 2 ABOUT 36" WOUND ON A $\frac{1}{4}D$. AXLE IN A 0.090" WIDE GAP - NEAR THE HAND AS IN "LOOP THE LOOP."

A NOTE ON A "FIGURE OF MERIT, $L\%$ ":

IN MONOGRAPH I (PAGES 4 AND 5), STRING TO Yo-Yo LEVERAGE $L\%$ IS DEFINED AS A RATIO OF WOUND RADIUS r_3 TO RADIUS OF GYRATION k_0 . IN "LOOP THE LOOP" PRACTICE, THIS PLAYER FINDS THAT LEVERAGE NEAR 70% PRODUCES COMFORTABLE 'FLAT' LOOPING. LEVERAGE NEAR 60% CAUSES THE Yo-Yo TO 'FLY', NEAR 80% CAUSES THE Yo-Yo TO 'SINK'.

Yo-Yo TO STRING COUPLING

EARLIER IT WAS STATED THAT A SLEEPING YO-YO AXLE AND SPINNING SIDES CAN ENGAGE A SLACKENED STRING. THE YO-YO MUST SPIN WITH ENOUGH STORED KINETIC ENERGY TO ENGAGE THE STRING POSITIVELY, REWINDING IT FIRMLY AND FULLY IN THE STRING GAP. THE ENERGY REQUIREMENT IS AT A MAXIMUM WHERE THE YO-YO MUST CLIMB THE STRING VERTICALLY TOWARD THE PLAYER'S HAND.

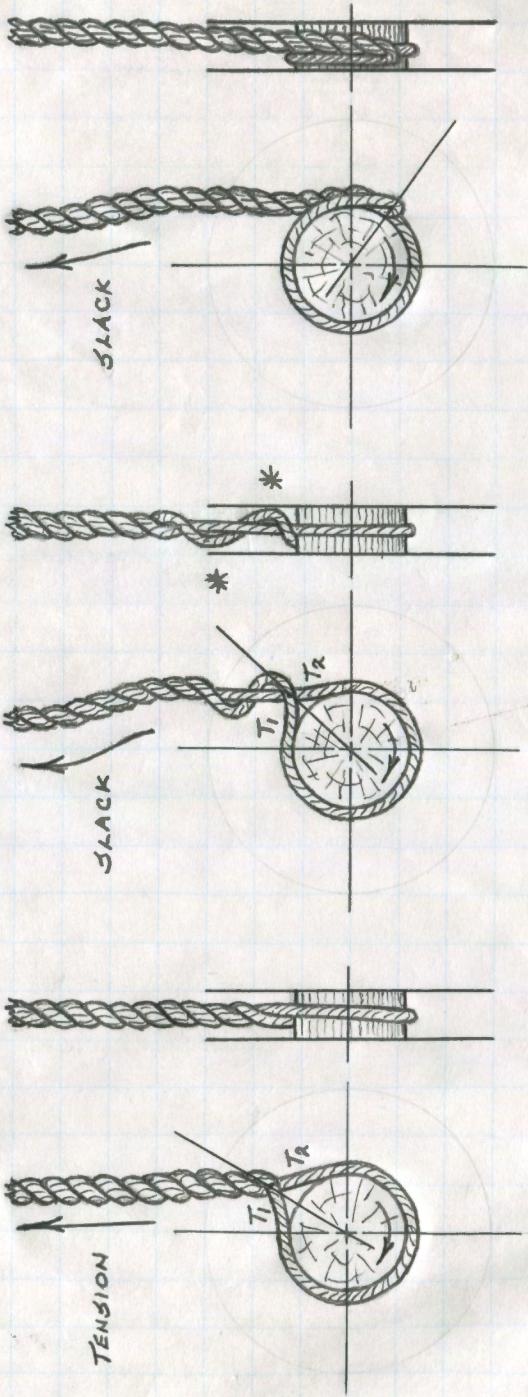
THE MECHANICS OF THIS DYNAMIC COUPLING ARE COMPLEX BUT WRAP, WEDGE, AND WIND ARE COMPONENT FUNCTIONS WHICH CAN BE DEFINED AND CONSIDERED SEPARATELY:

WRAP IS THE INITIAL WRAPPING OF STRING AT THE SURFACE OF THE SPINNING AXLE, LAYING DOWN ONLY ENOUGH STRING TO SPAN THE WIDTH OF THE STRING GAP.

WEDGE OCCURS AT WRAP COMPLETION WHERE INTIMATE LATERAL WEDGING CONTACT BEGINS BETWEEN THE STRING AND A SPINNING SIDE NEAR THE AXLE PERIPHERY.

WIND IS INDUCED UNDER STRING TENSION BY ANGULAR MOMENTUM OF THE SPINNING SIDES IN WEDGED CONTACT WITH THE STRING.

THE THREE FUNCTIONS ARE SEQUENTIALLY DEPENDENT; i.e. A FAILED WRAP LEADS TO A FAILED WEDGE AND FAILURE TO WIND.



SLEEPING $\gamma_0 - \gamma_0$ ————— \rightarrow SLACK INDUCED ————— \rightarrow TENSION RETURNING

THE SEQUENCED SKETCH GROUPS AT LEFT ATTEMPT TO PICTURE THE EARLY ENGAGEMENT OF THE YO-YO TO THE SLACKENED STRING.

THE SLEEPING YO-YO IS SHOWN IN THE EQUILIBRIUM STATE ANALYZED EARLIER. THE STRING IS UNDER TENSION AND $T_2 > T_1 > 0$.

ALL TENSIONS ARE REDUCED WITH STRING SLACK. RESIDUAL CONTACT OF THE LOOP ABOUT THE AXLE CAUSES SIGNIFICANT STRING DISTORTION AND $T_2 > 0 = T_1$. THREE ACTIONS HERE CAN AID THE WRAP FUNCTION: RESIDUAL AXLE CONTACT, INCIDENTAL CONTACT AT THE SPINNING SIDES (*), AND A SUBTLE WHIRLING BREEZE. THE BREEZE MAY BE SLIGHT, BUT SIGNIFICANT ESPECIALLY WHERE "STARBURSTS" OR OTHER ROUGH SURFACES ARE PRESENT IN THE STRING GAP NEAR THE AXLE.

IN THE FINAL SKETCH GROUP, THE SLACK STRING HAS BEEN DRAGGED FURTHER AROUND THE AXLE; CAPSTAN FRICTION NOW HAS NEW AND INCREASING EFFECT. VISUALIZE HERE THAT IN LESS THAN ANOTHER AXLE TURN THE ENTERING STRING WILL RUN OUT OF LATERAL ROOM IN THE STRING GAP AT THE AXLE SURFACE, COMPLETING THE WRAP.

GIVEN A POSITIVE AND SECURE WRAP THE WEDGE FUNCTION IS ASSURED AND, WITH ADEQUATE SPINNING MOMENTUM, THE WIND WILL BE FIRM AND COMPLETE.

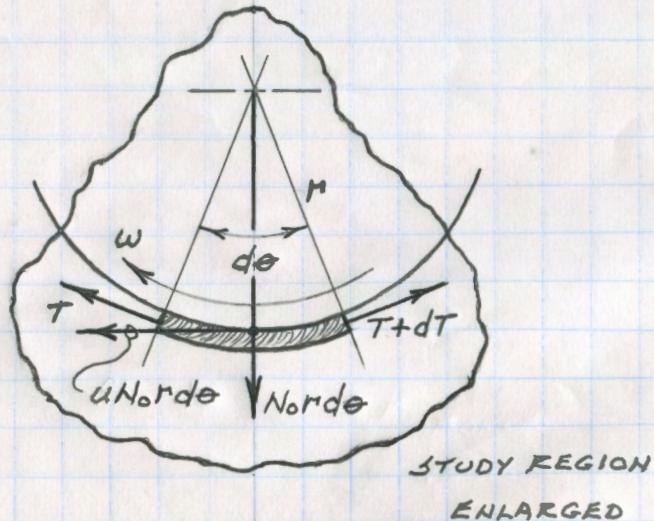
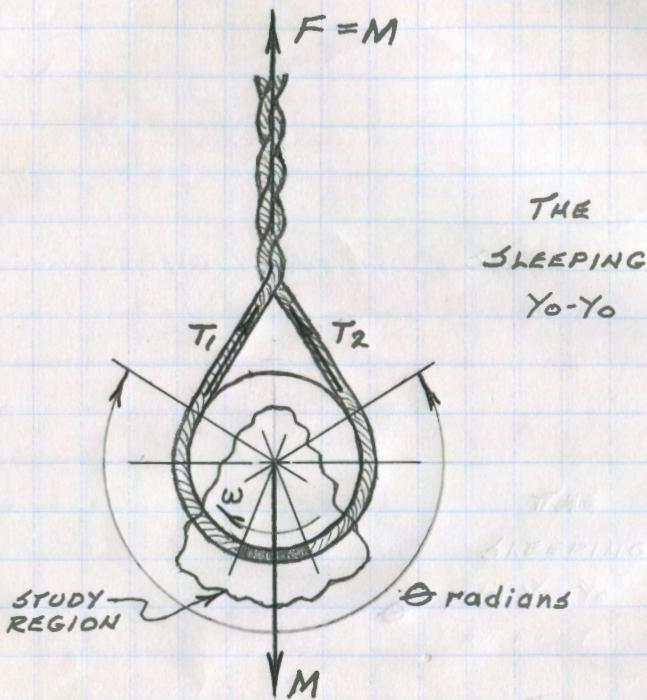
FIXED AXLE COUPLING RELIES ON SLIDING (DYNAMIC) CAPSTAN FRICTION AT THE AXLE TO STRING ARC OF CONTACT. COUPLING CAN BE DEFEATED BY A LOOSE STRING LOOP, A SLICK AXLE SURFACE, INADEQUATE SPIN VELOCITY, AND EVEN SLIGHT STRING TENSION INDUCED BY A SIDE BREEZE.

THE PERIPHERY OF A TRANSAXLE (SLEEVE OR BALL BEARING) IS HELD STATIONARY BY STATIC CAPSTAN FRICTION WITH THE STRING. LOW BEARING FRICTION ALLOWS VERY LONG SPIN DURATION; COMMON MEANS TO IMPROVE COUPLING RESPONSE INCLUDE:

- IN SLEEVE BEARINGS, AND IN SHIELDED OR OPEN BALL BEARINGS, VISCOUS LUBRICANT.
- SEALED BALL BEARINGS, WHERE THE SEALS INDUCE RUNNING TORQUE.
- WITH VERY LOW FRICTION BALL BEARINGS; GREASE IN, OR DEVICES ATTACHED TO, THE SPINNING SIDES - APPLYING TORQUE DIRECTLY TO THE BEARING PERIPHERY.
- MODIFICATIONS IN, OR ATTACHMENTS TO, THE SPINNING SIDES - APPLYING WRAPPING ACTION DIRECTLY TO THE SLACKENED AND DISTORTED (*) STRING; SEE THE SKETCHES.

TRANSAXLE YO-YO PERFORMANCE HAS BECOME "STATE-OF-THE-ART" IN RECENT YEARS. FIXED AXLE YO-YOS STILL REMAIN POPULAR; IN MANY SUCH YO-YOS TODAY "WOOD IS GOOD!"

THE CAPSTAN EQUATION



THE CAPSTAN EQUATION

THE SLEEPING YO-YO (SEE LEFT) ROTATES AT AN ANGULAR VELOCITY ω WITH ITS WEIGHT INDUCING FRICTION BETWEEN THE STRING AND THE AXLE PERIPHERY.

IN THE STUDY REGION, $d\theta$ IS A VERY SMALL INCREMENT OF θ , THE ANGLE OF CONTACT OF THE STRING AND AXLE. IN THE ANGLE $d\theta$, $r d\theta$ IS A LENGTH OF CONTACT DIRECTLY PROPORTIONAL TO THE AXLE RADIUS. THE TENSIONS T AND $T+dT$ IN THE STRAND EXERT A PRESSURE N_0 OVER THE LENGTH $r d\theta$, PRODUCING A NORMAL FORCE $N_0 r d\theta$. GIVEN μ , THE STRING TO AXLE COEFFICIENT OF FRICTION, THE FRICTION FORCE IS $\mu N_0 r d\theta$.

RADIAL VECTOR FORCES EQUILIBRIUM GIVES:

$$T \sin \frac{d\theta}{2} + (T + dT) \sin \frac{d\theta}{2} = N_0 r d\theta$$

WITH $d\theta$ VERY SMALL, $\sin \frac{d\theta}{2} = \frac{d\theta}{2}$. FURTHER, THE PRODUCT $dT \sin \frac{d\theta}{2}$ IS SMALLER STILL AND CAN BE IGNORED, LEAVING:

$$T \frac{d\theta}{2} + T \frac{d\theta}{2} = T d\theta = N_0 r d\theta \text{ AND,}$$

$$T = N_0 r$$

TANGENTIAL VECTOR FORCES EQUILIBRIUM GIVES:

$$(1) (T + dT) \cos \frac{d\theta}{2} - T \cos \frac{d\theta}{2} = u N_0 r d\theta$$

WITH $\frac{d\theta}{2}$ VERY SMALL, $\cos \frac{d\theta}{2} = 1$

$$dT \cos \frac{d\theta}{2} = dT = u N_0 r d\theta$$

EARLIER, $T = N_0 r$; ELIMINATING $N_0 r$ AND TRANSPOSING:

$$\frac{dT}{T} = u d\theta; \int \frac{dT}{T} = \int u d\theta$$

$$\ln T = u\theta + C; T = e^{u\theta + C}$$

USING THE CONSTANT OF INTEGRATION AS A COEFFICIENT*, EVALUATING T AT $\theta = 0$ radians:

$$T = C e^{u\theta}; T_0 = C e^0 = C \cdot 1 = C$$

$$T = T_0 e^{u\theta}$$

THE TENSION T_2 AT THE ANGLE θ MEASURED FROM THE POINT OF FIRST CONTACT WHERE $T_1 = T_0$ BECOMES FOR THE SLEEPING YO-YO:

$$T_2 = T_1 e^{u\theta}$$

$$* \text{ FROM } e^{u\theta + C} = e^{u\theta} \cdot e^C = C e^{u\theta}$$

THIS CAPSTAN EQUATION DERIVATION IS ADAPTED FROM "MECHANICS" BY J. P. DEN HARTOG* WHERE THE CAPSTAN IS IDENTIFIED AS A 'HOISTING DRUM'. THE EQUATION APPLIES TO BAND BRAKES AND BELT DRIVES ALSO. ON SAILING VESSELS, WIND-LOADED SAILS ARE HAULED IN AND 'PAYED OUT' WITH MODERATE EFFORT AND GOOD CONTROL USING ONE OR MORE TURNS OF LINE ON A POWERED CAPSTAN. WHERE THE LEVER IS A FORCE AMPLIFIER, THE CAPSTAN IS A TORQUE AMPLIFIER.

DIFFERENTIAL EQUATIONS OF THE FORM $\frac{dy}{t} = kdt$, WHERE k IS A CONSTANT OF PROPORTIONALITY, RELATE TO THE 'LAW OF NATURAL GROWTH AND DECAY' IN "CALCULUS" BY SMITH AND GRIFFIN.*

A HISTORICAL NOTE: VANNEVAR BUSH (1890-1974), EDUCATOR, SCIENTIST, AND INVENTOR, DEVELOPED THE FAMED DIFFERENTIAL ANALYZER AT MIT IN THE '30S. AN ANALOG COMPUTER, IT USED MANY TORQUE-AMPLIFYING 'INTEGRATORS', EACH BASED ON THE CAPSTAN EQUATION. DURING WORLD WAR II THE ANALYZER WAS USED TO GREAT RESEARCH ADVANTAGE, ONLY TO BECOME OBSOLETE WITH THE INTRODUCTION OF THE DIGITAL COMPUTER IN THE SAME PERIOD.

* SEE REFERENCES.

UNITS - AN ODD COUPLE

THE NEWTON AND THE KILOGRAM, WHICH MAY BE UNFAMILIAR TO SOME READERS, ARE IMPORTANT UNITS OF MEASURE IN THIS MONOGRAPH. IN "UNIVERSITY PHYSICS" (SEE REFERENCES) THEIR DEFINITIONS ARE FOUND:

"ONE NEWTON IS THE... FORCE THAT GIVES AN ACCELERATION OF ONE METER PER SECOND SQUARED TO... A MASS OF ONE KILOGRAM."

$$1 \text{ Newton} = 1 \text{ N} = 1 \frac{\text{kg} \cdot \text{m}}{\text{sec}^2}, \text{ AND}$$

$$1 \text{ N} = 0.225 \text{ lb}; 1 \text{ lb} = 4.45 \text{ N}$$

THE REFERENCE ALSO LISTS (PAGE 93):

"1 N = WEIGHT OF A MEDIUM APPLE"!

HISTORY TELLS OF ISAAC NEWTON'S FIRST THOUGHTS OF A "GRAVITY" IN NOTICING A FALLING APPLE. THE CONCEPT OF THE NEWTON UNIT DEFINITION - A VERY POWERFUL TOOL IN PHYSICAL ANALYSIS - IS REMARKABLE IN RELATION TO THE FALLING APPLE. IF, AS THE MYTH OF SOME UNKNOWN SCIENCE WAG HAS IT, THE APPLE "HIT NEWTON ON THE HEAD", PERHAPS WE SHOULD NOT TAKE THAT LITERALLY.

"... ONE RADIAN IS THE ANGLE SUBTENDED AT THE CENTER OF A CIRCLE BY AN ARC WITH A LENGTH EQUAL TO THE RADIUS OF THE CIRCLE.

... AN ANGLE IN RADIANS IS THE RATIO OF TWO LENGTHS, SO IT IS A PURE NUMBER, WITHOUT DIMENSIONS."

THE DEFINITION LEADS DIRECTLY TO THE FACT THAT IN ONE FULL CIRCUMFERENCE C OF A CIRCLE WITH RADIUS r , THERE MUST BE 2π RADIANS SINCE:

$$C = 2\pi \cdot r = \text{ONE REVOLUTION}$$

IN EVALUATING ANGULAR MOTION, THE "RATIO OF TWO LENGTHS" C/r FOR ONE REVOLUTION, USING METERS FOR MEASURE OF LENGTHS IS:

$$\frac{C}{r} = \frac{2\pi \cdot r}{r} \frac{\text{meters}}{\text{revolution}} = 2\pi \frac{\text{radians}}{\text{revolution}}$$

IN THIS MONOGRAPH:

- NEWTON MEASURE APPLICATION IS ESSENTIAL IN THE SPIN VELOCITY AND SPIN ENERGY STUDIES.

- RADIAN MEASURE APPLICATION APPEARS IN THE SLEEPING YO-YO, SPIN VELOCITY, SPIN ENERGY, AND THE CAPSTAN EQUATION STUDIES.

AUTHOR'S NOTE

"...THROWS OF 'EQUAL VIGOR' CITED AT PAGE 18 ARE ACCEPTABLE IN THE CONTEXT OF CONCEPTUAL MATHEMATICAL MODELLING, BUT THEN ONLY WITH REASONABLE FURTHER CONDITIONS. YO-YOS SO COMPARED MUST:

1. HAVE NEARLY EQUAL WEIGHT AND MOMENT OF INERTIA (SEE PAGE 20).
2. HAVE EQUAL STRING GAP WIDTH, USING STRING OF THE SAME TYPE AND LENGTH.

THE 'MODERN TRANSAXLE' (BC/TK YO-YOS SB2) GAP AND STRING WERE ADJUSTED TO ITEM 2. UNDER THESE CONDITIONS, THE FIXED AXLE TECHNIC AND THE TRANSAXLE SB2TM YIELDED NOMINAL SPIN DURATIONS OF 10 AND 60 SECONDS RESPECTIVELY. PLAYERS COMMONLY ACHIEVE THESE RESPECTIVE SPINS WITH MODERATE EFFORT, REACHING SIGNIFICANTLY LONGER SPINS USING GREATER EFFORT.

DIGITAL TACHOMETER (NON-CONTACT TYPE) MEASUREMENTS FOR THE RESPECTIVE 10 AND 60 SECOND SPINS RECORDED ROUGHLY EQUAL INITIAL VELOCITIES 4000 TO 5000 RPM.

HAPPY DAYS -

Don Watson



Captain Yo

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